Weak Invariants

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The notion of invariants plays an important role in various disciplines of sciences. Here, the concept of weak invariants is introduced. It is concerned mainly with dissipative systems and is defined so that the expectation values of weak invariants should be constant in time. This, in turn, characterizes the structure of the associated master equations. Although the expectation value of a weak invariant is constant in time by definition, its variation is not. A general formula for evolution of the variance is given in the case of the Fokker-Planck equation. As a simple example, the stochastic volatility model in finance is discussed. It is shown how increase of the variation can be suppressed by a proper choice of the initial condition, showing a prominent role of the weak invariant.